What is Discrete Mathematics?

- the study of **discrete** (as opposed to continuous) objects.
- Which (do you think) are discrete objects?
  
  characters?  digits?
  
  integers?  real numbers?
  
  set like {a, b, c}?  time?
  
  length?  Answer of ‘do you like me’?
What’s the difference?

- Discrete objects are **countable** (could be infinitely many)
  - we *can enumerate* elements of an infinite set \( S \) (**list** all elements of \( S \) **in a sequence**)

- Continuous objects are **not countable**
  - All real numbers in \([0, 1]\)
  - All ‘time’ in 1 second (time can be as small as possible)
  - The area within a circle
Examples of Discrete Objects

Set ← function → Set → integers

relation

logic

Prime numbers

counting

sequence

ordering

graph

tree

permutations

combinations
Examples of discrete objects

- integers
- steps taken by a computer program
- distinct paths to travel from point A to point B on a map along a road network
- ways to pick a winning set of numbers in a lottery
- ways to choose a password following specific rules
- number of valid Internet addresses
- possible paths between two cities using a transportation system
How much does it matter?

• provides the mathematical background needed for
  • all subsequent courses in computer science and
  • all subsequent courses in the many branches of discrete mathematics.
Some Problems Related to Discrete Mathematics (1/4)

- Can we find a path that crosses each bridge exactly once and returns to the starting point?

The 7 Bridges of Königsberg
Some Problems Related to Discrete Mathematics (2/4)

- Can we tile the following checkerboard using dominos?

Dominoes

Proof
Some Problems Related to Discrete Mathematics (3/4)

- From a standard deck of 52 cards, how many cards must be selected to guarantee that at least three cards of the same suit are chosen?

52 cards

4 suits
An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.

You go to the island and meet A and B.

- A says “B is a knight.”
- B says “The two of us are of opposite types.”

**Question**: What are the types of A and B?
Goals of a Course in Discrete Mathematics (1/3)

- **Mathematical Reasoning:**
  - ability to read, understand, and construct mathematical arguments and proofs.

- **Combinatorial Analysis:**
  - techniques for counting objects of different kinds.

- **Discrete Structures:**
  - abstract mathematical structures that represent objects (sets) and the relationships between them (relations, graphs, trees).
Goals of a Course in Discrete Mathematics (2/3)

- **Algorithmic Thinking:**
  - an algorithm is a *sequence of steps* that can be followed to solve any instance of a particular problem.

- Algorithmic thinking involves
  - specifying algorithms
  - analyzing the memory and time required by an execution of the algorithm
  - verifying that the algorithm will produce the correct answer.
Goals of a Course in Discrete Mathematics (3/3)

- **Applications and Modeling:**
  - appreciate and understand the wide range of applications of the topics in discrete mathematics
  - develop the ability to develop new models in various domains.

- **Concepts from discrete mathematics have been used**
  - to address problems in computing
  - to solve problems in many areas such as chemistry, biology, linguistics, geography, business, etc.
Discrete Mathematics is a Gateway Course

• Topics in discrete mathematics will be important in many courses that you will take in the future:
  • **Computer Science:** Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation, ......
Discrete Mathematics is a Gateway Course

- **Mathematics**: Logic, Set Theory, Probability, Number Theory, Abstract Algebra, Combinatorics, Graph Theory, Game Theory, Network Optimization, ...

- The concepts learned will also be helpful in continuous areas of mathematics.

- **Other Disciplines**: You may find concepts learned here useful in courses in philosophy, economics, linguistics, and other departments.
Course Enrollment

- There may be still some seats left.
- Max. 10% extra seats are available even though the class is full.
- These extra seats are reserved to students that are retaking this course.
- Qualified students should have my signature on the course adding form for approval.
Text Book

# Schedule (tentative)

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<td>The Foundations: Logic and Proofs (1/2)</td>
<td>10</td>
<td>Counting (1/2)</td>
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<tr>
<td>2</td>
<td>The Foundations: Logic and Proofs (2/2)</td>
<td>11</td>
<td>Counting (2/2)</td>
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<tr>
<td>3</td>
<td>Basic Structures: Sets, Functions, Sequences, Sums, and Matrices (1/2).</td>
<td>12</td>
<td>Advanced Counting Techniques (1/2)</td>
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<tr>
<td>4</td>
<td>Basic Structures: Sets, Functions, Sequences, Sums, and Matrices (2/2).</td>
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<td>Advanced Counting Techniques (2/2)</td>
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<td>5</td>
<td>Number Theory (1/2).</td>
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<td>Relations (1/2)</td>
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<td>6</td>
<td>Number Theory (2/2)</td>
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<td>7</td>
<td>Induction and Recursion (1/2)</td>
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<td>8</td>
<td>Induction and Recursion (2/2)</td>
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<td>Graphs (2/2)</td>
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<td>9</td>
<td>Review and Mid-term Exam.</td>
<td>18</td>
<td>Final Exam.</td>
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Scoring Policy

- (40%) Homework and quizzes.
  - Mostly self-study homework this year (2020).
    - 8 or 9 quizzes this year (2020).
- (30%) Mid-term exam.
- (30%) Final exam.
128 students in total
Statistics (Spring, 2018)

71 students in total
Statistics (Spring, 2019)

81 students in total
# Teaching Assistants (TAs)

<table>
<thead>
<tr>
<th>Name</th>
<th>E-mail</th>
<th>Phone</th>
<th>TA Time/Place</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albert</td>
<td><a href="mailto:albert860728@gmail.com">albert860728@gmail.com</a></td>
<td>ext. 56674</td>
<td>19:00-21:00 Tue. @ES703B</td>
</tr>
<tr>
<td>CHEN Feng Yang</td>
<td><a href="mailto:st9105323@gmail.com">st9105323@gmail.com</a></td>
<td>ext. 56674</td>
<td>13:00-15:00 Thu. @ES703B</td>
</tr>
</tbody>
</table>

ES: Microelectronics and Information Systems Research Center (電子與資訊研究中心)
Course Materials

- Slides are placed in new e3 system: https://e3new.nctu.edu.tw/
- All announcements are available in new e3 system: https://e3new.nctu.edu.tw/
- Companion website provided by the author of the textbook: http://www.mhhe.com/rosen
- Instructor’s e-mail: lhyen@nctu.edu.tw