1. Find all strings in \( L((a + b)^* b(a + ab)^*) \) of length less than four.
   \textbf{Ans.} \( b, ab, ba, bb, aab, bbb, abb, bab, baa, aba, bba \).

2. Find a regular expression for the set \( \{a^n b^m : (n + m) \text{ is even} \} \).
   \textbf{Ans.} \((aa)^*(bb)^* + a(aa)^*b(bb)^*\).

3. What language do the expression \( (\emptyset)^* \) and \( a\emptyset \) denote?
   \textbf{Ans.} \( L((\emptyset)^*) = (L(\emptyset))^* = \{\emptyset\}^* = \{\emptyset\}, L(a\emptyset) = L(a)L(\emptyset) = \emptyset \).

4. Find a regular expression for \( L = \{w \in \{0, 1\}^* : w \text{ has exactly one pair of consecutive zeros} \} \).
   \textbf{Ans.} \((1 + 01)^*00(1 + 10)^*\).

5. Let \( \Sigma = \{a, b, c\} \). Give a regular expression for the all strings containing no more than three \( a \)’s.
   \textbf{Ans.} \((b+c)^* + (b+c)^*a(b+c)^* + (b+c)^*a(b+c)^*a(b+c)^* + (b+c)^*a(b+c)^*a(b+c)^*a(b+c)^*\).

6. Let \( \Sigma = \{0, 1\} \). Write a regular expression for all strings not ending in 01.
   \textbf{Ans.} \((\lambda + 0 + 1 + (0 + 1)^*)(00 + 10 + 11)\).

7. Find an NFA that accepts the language \( L(ab^* aa + bba^* b^*) \).
   \textbf{Ans.} The following graph represents the NFA \( M = (\{q_0, q_1, \ldots, q_9\}, \{a, b\}, \delta, q_0, \{q_9\}) \) that accepts \( L(ab^* aa + bba^* b^*) \), where \( \delta \) is described as in the graph.

![NFA Diagram]

8. Find a DFA that accepts \( L = L(ab^* a^*) \cap L((ab)^* ba) \).
   \textbf{Ans.} \( L = L(ab^* a^*) \cap L((ab)^* ba) = \{abba\} \). The following graph represents the DFA \( M = (\{q_0, q_1, \ldots, q_5\}, \{a, b\}, \delta, q_0, \{q_4\}) \) that accepts \( L \), where \( \delta \) is described as in the graph.

![DFA Diagram]
9. Find the minimal DFA that accepts $L(a^*bb) \cup L(ab^*ba)$.

**Ans.** The following graph represents an NFA $M_N = (\{q_0, q_1, \ldots, q_6\}, \{a, b\}, \delta, q_0, \{q_6\})$ that accepts $L(a^*bb) \cup L(ab^*ba)$, where $\delta$ is described as in the graph.

We then transform $M_N$ into a DFA $M_D = (\{q_{012}, q_{14}, q_1, q_3, q_{345}, q_{456}, q_6, q_{45}\}, \{a, b\}, \delta, q_{012}, \{q_6, q_{456}\})$ that accepts $L(a^*bb) \cup L(ab^*ba)$, where $\delta$ is described as in the graph.

By using the mark procedure, we can finally partition the state set as $\{q_{012}\}$, $\{q_{14}\}$, $\{q_1\}$, $\{q_3\}$, $\{q_{345}\}$, $\{q_{456}\}$, $\{q_6\}$, $\{q_{45}\}$, and $\{q_x\}$. Thus, such a DFA is minimal.

10. Find a regular expression for the language accepted by the following automata.

**Ans.** $a^*ba^*$.

11. Find a regular grammar that generates the language $L(aa^*(ab + a)^*)$.

**Ans.** The grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ with productions $S \rightarrow aA$, $A \rightarrow aA|B$, $B \rightarrow abB|aB|\lambda$.

12. Find a regular grammar that generates the set of all real numbers in $\mathbb{C}$.

**Ans.** The grammar $G = (\{S, A, B, C, D\}, \{+, -, ., 0, 1, \ldots, 9\}, S, P)$ with productions $S \rightarrow +((0 + 1 + \ldots + 9)A - (0 + 1 + \ldots + 9)A)A | (0 + 1 + \ldots + 9)A | (0 + 1 + \ldots + 9)B$, $B \rightarrow .C$, $C \rightarrow [0 - 9]C$.