DCP 3122 Introduction to Formal Languages

Homework #2, due 4/6/2012 (Friday)

1. Write regular expressions for the following regular languages:
   (a) \( L_1 = \{a^n b^m : n \geq 4, m \leq 3\} \)
   (b) \( L_1 = \{a^n b^m : n+m \text{ is even}\} \)
   (c) All strings containing no more than three a’s.
   (d) All strings containing an even number of a’s.

2. Find NFA’s for the following regular expressions:
   (a) \( L(aa^*aba^*b^*) \)
   (b) \( L((aa^*)b)^* \)

3. Consider the following generalized transition graph.
   (a) Find an equivalent generalized transition graph with only two states for the above graph.
   (b) Find the regular expression for the above generalized transition graph.
   (c) What is the language accepted by this graph?

4. Find a regular grammar that generates the language \( L(aa^*(ab+a)^*) \).

5. Use the construction in Theorem 4.1 to find NFA for \( L((a+b)a^*) \cap L(baa^*) \).

6. Show that if \( L \) is a regular language, so is \( L_1=\{v_1u v_2 : u \in L, |v_1|=|v_2|=1\} \).

7. Show that if \( L \) is a regular language, so is \( \text{tail}(L)=\{y : xy \in L \text{ for some } x \text{ in } \Sigma^*\} \).
   \textbf{Hint:} Assume that DFA \( M=(Q, \Sigma, \delta, q_0, F) \) accepts \( L \). Then, try to specify an NFA/DFA for \( \text{tail}(L) \) formally.

8. Show that there is an algorithm of determining whether or not \( L_1 \subseteq L_2 \), where \( L_1 \) and \( L_2 \) are given.

9. Show that the following languages are not regular.
(a) \(L=\{ww : w \in \{a,b\}^*\}\)
(b) \(L=\{a^n b^k : n<k\}\)
(c) \(L=\{a^n b^k : |n-k|=2\}\)
(d) \(L=\{w \in \{a,b,c\}^* : n_a(w)=n_b(w)+n_c(w)\}\)
(e) \(L=\{a^p : p \text{ is prime}\}\)

10. Prove or disprove that \(L_1\) and \(L_2\) are both non-regular, neither is \(L_1 \cup L_2\)