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Statistical Mapping of Cortical Activities using Minimum-Variance Maximum-Discrimination Spatial Filtering

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Abstract—**This paper presents a new spatial filtering technique for statistical mapping of neuronal sources by using magnetoencephalography data. In addition to the unit-gain constraint and the minimum-variance criterion that can reconstruct the activation magnitude of the targeted source while suppressing the contribution from other sources, the proposed technique exploits a maximum-discrimination criterion that can maximize the discrepancy between the reconstructed neuronal activities in the control (or resting) state and those in the active state. Imposing the maximum-discrimination criterion leads to a closed-form solution of the source orientation, which can then be used to compute the proposed minimum-variance maximum-discrimination spatial filter for each probed position. When applied to a finger-lifting study, F-statistic map computed from the reconstructed neuronal activities on the cortical surface clearly identify the sensorimotor area with high contrast.**[†]

Keywords—**Spatial filter, maximum discrimination, neuromagnetic imaging, beamformer, synthetic aperture magnetometry**

I. Introduction

Spatiotemporal brain activation imaging facilitates functional brain research in both clinical and basic neuroscience area. Compared with other indirect brain imaging technologies such as positron emission tomography (PET) and functional magnetic resonance imaging (fMRI), which image metabolic or hemodynamic consequences, magnetoencephalography (MEG), a direct measure of brain activity induced by electrophysiological events, is characteristic of higher temporal resolution [1]. To estimate the neuronal activities from the MEG recordings, various kinds of source modeling and estimation methods have been developed in the literature [2, 3]. Among them dipole fitting is widely used due to its simplicity and robustness. The drawback of this method is the requirement to assign the number of dipoles beforehand. Distributed source estimation methods, such as minimum-norm estimation and minimum-current estimation, have the problem of tending to identify superficial sources [4].

Beamforming, a kind of spatial filtering technique, is an emerging method for solving MEG inverse problem [4, 5, 6, 7]. By probing the source space in a voxel-by-voxel manner, the spatial filter is individually calculated for reconstructing the neuronal activation magnitude for each position. Furthermore, orientation of the dipole source has to be determined either by nonlinear search [6] or by decomposing into three orthogonal components [7]. In this work, we develop a novel spatial filtering technique for statistical mapping of neuromagnetic sources. Based on the unit-gain constraint and the minimum-variance criterion, as in conventional beamformers, the activation magnitude of the targeted source can be reconstructed while suppressing the contribution from other sources. Moreover, our method exploits an additional criterion that can maximize the discrimination between the reconstructed neuronal activities in the control (or resting) state and those in the active state. By imposing this criterion, a closed-form solution of the source orientation is derived such that the spatial filter can be obtained very efficiently for each targeted position. Once the neuronal activity waveform is reconstructed in the source space, F-statistic map can be calculated to reveal cortical regions with significant difference of activities between control and active states.

II. Methods

A. Overview

Fig. 1 depicts the schema of this work. We first obtain the 3-D polygon mesh of the cortical surface, on which source activation is estimated, by using a semi-automated cortex segmentation tool, SureFIT [8], from the MRI volume. MEG signal preprocessing procedure includes eye blinking rejection, synchronized averaging, signal space projection, and band-pass filtering. Then, the source strength for each vertex on the cortical surface can be reconstructed through the calculated spatial filters, whose design will be elaborated below. Finally, the statistical inference of cortical regions with strong activation can be drawn by calculating F-statistic map from the reconstructed activities.

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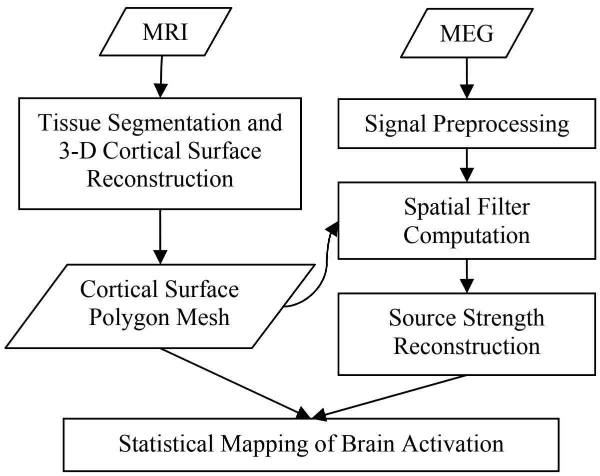

Fig. 1. Flowchart of the proposed method.

B. Forward Model

Source strength reconstruction, which is an inverse problem, needs a forward solution involving the prediction of magnetic fields resulting from primary currents [9]. Based on the spherical head model, which assumes that head is composed of concentric spheres with homogeneous and isotropic conductivity, MEG forward problem has a closed form solution $\mathbf{B}(\mathbf{r})$ for magnetic fields measured at sensor location $\mathbf{r}$:

$$\mathbf{B}(\mathbf{r})=\frac{\mu_0}{4\pi F^2(\mathbf{r},\mathbf{r}_q)}\left(F(\mathbf{r},\mathbf{r}_q)\mathbf{q}\times\mathbf{r}_q-(\mathbf{q}\times\mathbf{r}_q\cdot\mathbf{r}\nabla F(\mathbf{r},\mathbf{r}_q))\right), \quad (1)$$

where $F(\mathbf{r}, \mathbf{r}_q)$ is a scalar function,

$$F(\mathbf{r},\mathbf{r}_q)=d(\mathbf{r}d+\mathbf{r}^2-(\mathbf{r}_q\cdot\mathbf{r})),$$

$\nabla F(\mathbf{r}, \mathbf{r}_q)$ is a vector function,

$$\nabla F(\mathbf{r},\mathbf{r}_q)=\left(\frac{d^2}{\mathbf{r}}+\frac{\mathbf{d}\cdot\mathbf{r}}{d}+2d+2\mathbf{r}\right)\mathbf{r}-\left(d+2\mathbf{r}+\frac{\mathbf{d}\cdot\mathbf{r}}{d}\right)\mathbf{r}_q,$$

$\mathbf{r_q}$ denotes the equivalent current dipole location, $\mathbf{q}$ denotes the dipole moment, and

$$\mathbf{d} = \mathbf{r} - \mathbf{r_q};\ \mathrm{d} = |\mathbf{d}|.$$

C. Spatial Filter Design

For each dipole source, a spatial filter is a weighting function mapping the measurements in sensor space to the source strength. Given a spatial filter $\mathbf{w}$ and measurements $\mathbf{m}(t)$, the source signal $\mathbf{s}(t)$ can be reconstructed by

$$\mathbf{s}(t)=\mathbf{w}^{\mathrm{T}}\mathbf{m}(t), \quad (2)$$

and the signal power p over a period of n time points can be calculated by

$$p=\frac{1}{n}\sum_{t=1}^{n}\left\{\mathbf{w}^{\mathrm{T}}\mathbf{m}(t)\right\}^2. \quad (3)$$

For the purpose of maintaining the signal strength originating at a specific source location after filtering, the desired spatial filter must be subjected to the linear constraint,

$$\mathbf{w}^{\mathrm{T}}\mathbf{l}=1, \quad (4)$$

where $\mathbf{l}$ is the *leadfield*, the forward solution (1) of unit moment current dipole. The leadfield is the product of the gain matrix $\mathbf{G}$ and the dipole orientation vector $\mathbf{j}$ [2, 3]:

$$\mathbf{l}=\mathbf{G}\mathbf{j}. \quad (5)$$

Under the linear constraint there are infinite solutions (spatial filters) in the solution space of the source strength estimation problem. Consequently, a spatial filter subject to additional appropriate constraints may provide better results. In the following, we present the deduction of the proposed minimum-variance maximum-discrimination spatial filtering technique.

1. Minimum-norm spatial filter ($\mathbf{w}_{\mathrm{n}}$)‡

Among the spatial filters subject to linear constraint of (4), the minimum norm spatial filter is calculated from the leadfield $\mathbf{l}$:

$$\mathbf{w}_{\mathrm{n}}=\frac{\mathbf{l}}{\|\mathbf{l}\|^2}. \quad (6)$$

Because MEG recordings are not involved, this kind of spatial filter can be calculated beforehand. Furthermore, the noise sensitivity is minimized due to the minimum norm of the spatial filter. Poor spatial specificity is its chief shortcoming.

2. Minimum-norm minimum-variance spatial filter ($\mathbf{w}_{\mathrm{nv}}$)

Given both dipole position and orientation of a specific neuronal source, one can obtain the leadfield $\mathbf{l}$. To retain the strength of this specific source while attenuating the contributions from other sources, the minimum variance beamformer [5, 6, 7] is given by

$$\mathbf{w}_{\mathrm{nv}}=\frac{(\mathbf{C}+\alpha\lambda\mathbf{I})^{-1}\mathbf{l}}{\mathbf{l}^{\mathrm{T}}(\mathbf{C}+\alpha\lambda\mathbf{I})^{-1}\mathbf{l}}, \quad (7)$$

where $\mathbf{C}$ is the covariance matrix of MEG measurements, λ is the maximum eigenvalue of $\mathbf{C}$, and α is the regularization parameter used for the tradeoff between spatial specificity and noise sensitivity.

3. Minimum-variance maximum-discrimination spatial filter ($\mathbf{w}_{\mathrm{vD}}$)

In order to determine the dipole orientation $\mathbf{j}$, another criterion or constraint is required. Here we adopt the Fisher criterion [10] to maximize the discrimination of the

‡ Upper- and lower-case letters in the spatial filter abbreviation denote "maximum" and "minimum" respectively. For example, $\mathbf{w}_{\mathrm{vD}}$ represents the minimum-variance maximum-discrimination spatial filter.

reconstructed activities between the control and active states. Substituting (5) into (7) yields

$$\mathbf{w}_{\mathrm{vD}} = \frac{(\mathbf{C}+\alpha\lambda\mathbf{I})^{-1}\mathbf{Gj}}{\mathbf{j}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}(\mathbf{C}+\alpha\lambda\mathbf{I})^{-1}\mathbf{Gj}} = \frac{\mathbf{Aj}}{\mathbf{j}^{\mathrm{T}}\mathbf{Bj}}, \quad (8)$$

where **A** stands for $(\mathbf{C}+\alpha\lambda\mathbf{I})^{-1}\mathbf{G}$, and **B** for $\mathbf{G}^{\mathrm{T}}\mathbf{A}$.

Based on the Fisher criterion, the dipole orientation **j** can be obtained by determining the projection axis, which is the spatial filter $\mathbf{w}_{\mathrm{vD}}$ here, that maximizes the projected between-class scatter divided by the projected within-class scatter:

$$\hat{\mathbf{j}} = \arg\max_{\mathbf{j}} \frac{\mathbf{w}_{\mathrm{vD}}^{\mathrm{T}}\mathbf{S}_{\mathrm{B}}\mathbf{w}_{\mathrm{vD}}}{\mathbf{w}_{\mathrm{vD}}^{\mathrm{T}}\mathbf{S}_{\mathrm{W}}\mathbf{w}_{\mathrm{vD}}} = \arg\max_{\mathbf{j}} \frac{\mathbf{j}^{\mathrm{T}}(\mathbf{A}^{\mathrm{T}}\mathbf{S}_{\mathrm{B}}\mathbf{A})\mathbf{j}}{\mathbf{j}^{\mathrm{T}}(\mathbf{A}^{\mathrm{T}}\mathbf{S}_{\mathrm{W}}\mathbf{A})\mathbf{j}} = (\mathbf{A}^{\mathrm{T}}\mathbf{S}_{\mathrm{W}}\mathbf{A})^{-1}\left[\mathbf{A}^{\mathrm{T}}(\overline{\mathbf{m}}_{\mathrm{a}} - \overline{\mathbf{m}}_{\mathrm{c}})\right], \quad (9)$$

$$\overline{\mathbf{m}}_{\mathrm{a}} = \frac{1}{n_a}\sum_{t=1}^{n_a}\mathbf{m}_{\mathrm{a}}(t), \quad (10)$$

$$\overline{\mathbf{m}}_{\mathrm{c}} = \frac{1}{n_c}\sum_{t=1}^{n_c}\mathbf{m}_{\mathrm{c}}(t), \quad (11)$$

$$\mathbf{S}_{\mathrm{W}} = \sum_{t=1}^{n_a}(\mathbf{m}_{\mathrm{a}}(t) - \overline{\mathbf{m}}_{\mathrm{a}})(\mathbf{m}_{\mathrm{a}}(t) - \overline{\mathbf{m}}_{\mathrm{a}})^{\mathrm{T}} + \sum_{t=1}^{n_c}(\mathbf{m}_{\mathrm{c}}(t) - \overline{\mathbf{m}}_{\mathrm{c}})(\mathbf{m}_{\mathrm{c}}(t) - \overline{\mathbf{m}}_{\mathrm{c}})^{\mathrm{T}}, \quad (12)$$

$$\mathbf{S}_{\mathrm{B}} = (\overline{\mathbf{m}}_{\mathrm{a}} - \overline{\mathbf{m}}_{\mathrm{c}})(\overline{\mathbf{m}}_{\mathrm{a}} - \overline{\mathbf{m}}_{\mathrm{c}})^{\mathrm{T}}, \quad (13)$$

where $\mathbf{S}_{\mathrm{B}}$ and $\mathbf{S}_{\mathrm{W}}$ are the between- and within-class scatter matrices, $\overline{\mathbf{m}}_{\mathrm{a}}$ and $\overline{\mathbf{m}}_{\mathrm{c}}$ are the average measurement data of active and control states with period of n_a and n_c time points, respectively.

Substituting **j** in (8) with $\hat{\mathbf{j}}$ in (9) results in the linearly constrained, minimum norm, minimum variance, and maximum discriminant beamformer (14), which is a closed form solution maximizing Fisher's criterion without searching for dipole orientation.

$$\mathbf{w}_{\mathrm{vD}} = \frac{\mathbf{A}\hat{\mathbf{j}}}{\hat{\mathbf{j}}^{\mathrm{T}}\mathbf{B}\hat{\mathbf{j}}}. \quad (14)$$

D. Statistical Mapping of Neuronal Activities

Since the forward solution, or the leadfield, is inversely-squared-proportional to the dipole location [9], leadfield norm of deeper source is smaller than that of superficial ones. Consequently, the norm of the computed spatial filter as well as the reconstructed source strength is larger for deeper sources due to the unit-gain constraint. Hence, to obtain a meaningful statistical brain activation mapping, F-statistics ρ is calculated as follows:

$$\rho = \frac{p_{\mathrm{a}}}{p_{\mathrm{c}}}, \quad (15)$$

where p_{a} and p_{c} are the reconstructed signal powers in the active and control states, respectively, calculated by (3).

When the spatiotemporal cortical activity is required, it can be obtained by

$$\rho_t = \frac{p_{\mathrm{w}_t}}{p_{\mathrm{c}}} \quad (16)$$

where p_{w_t} is the signal power over a short period of time.

III. Experiments

The movement-related magnetic fields of one right-handed healthy subject were measured by a whole-head 306-channel neuromagnetometer (Vectorview, Neuromag Ltd., Finland). The subject was asked to sit in a comfortable chair with eyes open in a magnetically shielded room. The subject performed self-paced, brisk finger extension (finger lifting) movements at irregular time intervals longer than 8 sec. Finger extension was followed immediately by brief muscle relaxation. The commencement of finger movement was registered using an optical pad and trigger time was defined as zerotime. About 100 epochs of MEG measurements with a sampling rate of 250 Hz were acquired and averaged according to the trigger onsets. The average measurements were analyzed by using the above mentioned spatial filters.

The individual MRI images of size 256x256x128 obtained with a Siemens MR scanner where the MR-RAGE pulse sequence was performed with TR = 1800 ms, TE = 4.38ms, TI = 1100 ms, FOV = 230x230x192mm^3. The MEG device and the MR coordinate systems were co-registered by three landmarks (left pre-auricular, right pre-auricular, and nasion).

As shown in Fig. 2, the F-statistics, which was calculated by (15) and labeled as *strength index* for the three kinds of spatial filter mentioned above, is rendered on the segmented 3-D cortical surface mesh of the left hemisphere (containing 69530 vertices). The periods of the active and control states were selected from 0s to 0.5s and from -3s to -2.5s, respectively. Apparently, the position with significant F-statistics is located in the hand region of the primary sensorimotor cortex. However, the $\mathbf{w}_{\mathrm{n}}$ filter in (6), due to the worst spatial specificity between the three filters, produces a less-focal distribution, whereas the $\mathbf{w}_{\mathrm{nv}}$ filter in (7) yields a focal yet rough one. The $\mathbf{w}_{\mathrm{vD}}$ filter in (14) gives a focal and smooth result by computing dipole orientation without constrained to segmented surface normals. This advantage even helps the $\mathbf{w}_{\mathrm{vD}}$ filter produce an even more focal, yet

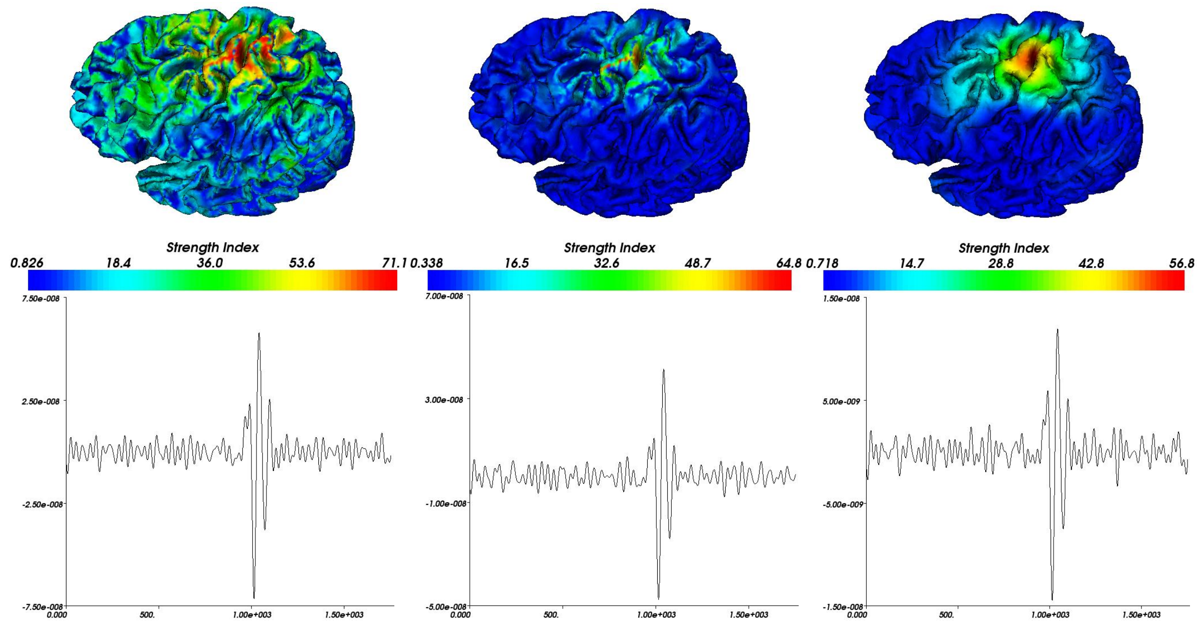

Fig. 2. The estimated F-statistics (top) and reconstructed signal wave of the maximum strength cortex (bottom). Columns, from left to right, are the results of applying the $\mathbf{w}_n$ filter (6), the $\mathbf{w}_{nv}$ filter (7), and the $\mathbf{w}_{vD}$ filter (14), respectively. In the latter two filters, α is set to 0.3.

smooth still, with α set to 0.003, under which the $\mathbf{w}_{nv}$ filter would provide messy results.

V. Conclusion

The presented minimum variance maximum discrimination MEG beamformer suggests a fast and robust neuromagnetic source imaging technique for clinical and neuroscience studies. The F-statistic map computed from the reconstructed neuronal activities on the cortical surface clearly identify the sensorimotor area with high contrast.

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