Route Throughput Analysis with Spectral Reuse for Multi-Rate Mobile Ad Hoc Networks

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Abstract

The mobile ad hoc networks (MANETs) have received a lot of attention for its flexible network architecture. While many routing protocols have been proposed for MANETs based on different criteria, few have considered the impact of multi-rate communication capability that is supported by many current WLAN products. Given a routing path, this paper provides an analytic tool to evaluate the expected throughput of the route with spectral reuse, assuming that hosts move following the discrete-time, random-walk model. The derived result can be added as another metric for route selection. Simulation results show that the proposed formulation can be used to evaluate path throughput accurately.

Keywords: Ad hoc networks, mobile computing, mobile networks, routing, wireless communication, spectral reuse.

I. INTRODUCTION

The mobile ad hoc network (MANET) is a flexible and dynamic architecture that is attractive due to its ease in network deployment. Routing is perhaps one of the most intensively addressed issues in MANET. Many different criteria have been used in route selection, including hop count [5], signal strength [11], route lifetime [3], and energy constraint [12]. Among these metrics, hop count may be the most widely used metric in choosing routes. When a hop-count based routing protocol is given multiple paths, the shortest path is normally selected and a random path is selected when there is a tie. This metric has the advantage of simplicity, requiring no additional measurements and incurring the least number of transmissions. The primary disadvantage of this metric is that it does not take packet loss or available bandwidth into account, especially when network interfaces can transmit at multiple rates [10]. It has been shown in [4] that a route which minimizes the hop count does not necessarily maximize the throughput of a flow.

While it is true that there is no single route selection metric that is able to best fit all possible routing scenarios in MANET, few works have considered the impact of multi-rate communication capability that is widely supported by many current wireless LAN products. For example, IEEE 802.11b supports rates of 11, 5.5, 2, and 1 Mbps, while IEEE 802.11a supports rates of 6, 9, 12, 18, · · · , and 54 Mbps. Route selection is more complicated in a multi-rate MANET than in a single-rate environment. Also, there exists an inherent tradeoff between transmission rates and their effective transmission ranges [2]. To support reliable data transmissions, longer-range communications must use lower rates, and vice versa. Auto-rate selection protocols [1], [6] do exist at the link level. Reference [13] proposes a multi-rate-aware topology control algorithm to enhance the network throughput in multi-hop ad hoc networks, and [14] uses fast links (with high nominal bit rate) to improve the system throughput in wireless mesh networks. However, they only focus on static network environment without taking mobility into account. Reference [17] proposes a multi-rate-aware sub-layer between the MAC and the network layers to improve resource utilization.

and to minimize power consumption, but the effect of multi-rate communications at the routing level is not yet fully addressed.

In this paper, we consider a MANET where each wireless link can support multiple rates and has the auto-rate selection capability. Given a routing path, this paper provides an analytic tool to evaluate the expected throughput of the route with spectral reuse, assuming that hosts move following the discrete-time, random-walk model. The result can be added as a new metric for route selection in MANET. (We comment that we do not intend to propose a new routing protocol here. But the proposed results may be used in many current protocols to compute a new route selection metric.)

The rest of this paper is organized as follows. Section 2 presents our system model. Section 3 shows our analysis results. Simulation results are presented in Section 4. Section 5 concludes this paper.

II. SYSTEM MODEL

In this paper, we assume that each mobile host roams around in the network area following the discrete-time, random-walk mobility model, which has been widely used in several works [7], [8], [9]. In this model, the network area is partitioned into a number of hexagonal cells, each with radius $r$ and each with a coordinate $(x, y)$, as shown in Fig. 1(a). Cells on the $x$-axis are numbered $(x, 0)$, and those on the $y$-axis $(0, y)$. The coordinates of other cells are obtained by mapping them onto these two axes, as is normally done in the Cartesian coordinate system.

Although hosts actually roam around in continuous time domain, we will work in discrete time domain by dividing time into fixed-length units. We assume that mobile hosts roam around in a cell-to-cell basis following the random walk model. Given a mobile host at any cell, it will move into any one of its six neighboring cells in the next time unit with an equal probability of $1/6$.

Cells in the network are further divided into layers as follows. Cell $(0, 0)$ is on the layer-0. The six neighboring cells of cell $(0, 0)$ are the layer-1 cells, and the outer cells surrounding layer-$i$ cells are said to be on layer $(i + 1)$. The number of cells included in an $n$-layer network is $3n^2 + 3n + 1$. In this paper, we will model the transmission range of a mobile host by a certain number of layers, by assuming its current location at layer 0.

Following [16], we use a vector to represent the state of a wireless link. Specifically, given a wireless link between two hosts located at cells $(x, y)$ and $(x', y')$, we represent the link’s state as a vector $\langle x' - x, y' - y \rangle$. A routing path thus may contain a sequence of vectors, each representing a wireless link. For example, a routing path containing hosts in cells $(0, 1)$, $(3, 1)$, and $(7, -3)$ in the order can be written as $[(3, 0), (4, -4)]$. 

![Fig. 1. (a) a cellular system to model station mobility, and (b) the “folding” of link states.](image-url)
Fig. 2. Example of link state changes.

### TABLE I

<table>
<thead>
<tr>
<th>State Change</th>
<th>Probability</th>
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<tr>
<td>(\langle x, y \rangle \rightarrow \langle x-1, y+1 \rangle)</td>
<td>2/36</td>
</tr>
<tr>
<td>(\langle x-1, y \rangle \rightarrow \langle x+1, y-1 \rangle)</td>
<td>2/36</td>
</tr>
<tr>
<td>(\langle x, y-1 \rangle \rightarrow \langle x+2, y-2 \rangle)</td>
<td>1/36</td>
</tr>
<tr>
<td>(\langle x-1, y \rangle \rightarrow \langle x-1, y+1 \rangle)</td>
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<td>(\langle x, y+1 \rangle \rightarrow \langle x, y+2 \rangle)</td>
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<td>(\langle x-2, y \rangle \rightarrow \langle x-2, y+1 \rangle)</td>
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<td>(\langle x-2, y \rangle \rightarrow \langle x-2, y+2 \rangle)</td>
<td>1/36</td>
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Based on the random walk model, we can derive a probability model for the state change of a wireless link. Let \(\langle x, y \rangle\) be the state of a wireless link connecting two neighboring hosts at time \(t\). At time \(t+1\), each of the hosts may move into one of its six neighboring cells with probability of \(1/6\). This gives 36 combinations of the two hosts’ next locations (as shown in Fig. 2), which can be reduced to 19 link states with different probabilities (as shown in Table I) [16].

Suppose that the transmission distance of a host is \(n\) layers. Then the number of states for a wireless link will be as large as \(3n^2 + 3n + 1\). To prevent the problem of state explosion that so many states need to be taken into consideration, [15] proposes to merge equivalent cells by “folding” the 12 sectors in Fig. 1(b) into one (cells of the same indices are equivalent). This reduces the number of states by around \(1/12\). Detailed derivations can be found in [15]. We will adopt the state reduction in this paper.

Most current wireless LAN cards support automatic rate selection depending on channel conditions. For example, IEEE 802.11b standard supports four transmission rates: 11, 5.5, 2, and 1 Mbps. When the MAC layer overheads are taken into account (control overheads, contention overheads, collision costs, etc.), the effective link rates may be reduced from 11, 5.5, 2, and 1 Mbps to 4.55, 3.17, 1.54, and 0.85 Mbps, respectively [2]. We assume that the rate of a wireless link will depend on the distance between the two hosts of the link. Reference [2] provides a general theoretical model of the attainable throughput in multi-rate ad hoc wireless networks.

### III. Route Throughput Analysis

A route consists of a number of wireless links. Given a routing path, our goal is to determine the expected route throughput based on the random walk model. In the previous section, we have derived how a wireless link changes states. Suppose that each mobile host has a transmission range of \(n\) layers. Then we can model a wireless link by considering an \((n + 2)\)-layer network. For example, Fig. 3 shows the state transition diagram of a wireless link when \(n = 5\). Note that states \(\langle 6, 0 \rangle, \langle 5, 1 \rangle, \langle 4, 2 \rangle, \langle 3, 3 \rangle, \langle 7, 0 \rangle, \langle 6, 1 \rangle, \langle 5, 2 \rangle, \text{ and } \langle 4, 3 \rangle\) are “absorbing” states such that \(x + y > n\) for state \(\langle x, y \rangle\), which means the distance between mobile hosts is larger than the transmission range and once a wireless link changes to any of these states, the link is considered broken.

The state transition probability of a wireless link in Fig. 3 can be modeled by a matrix \(M\) in Fig. 4, where each element \(M_{i,j}\) represents the probability for a link to transit from state \(i\) to state \(j\). \(M^k\) is the \(k\)-th power of \(M\), which represents the state transition probabilities after \(k\) time units. That is, \(M^k_{i,j}\) is the probability that a link at state \(i\) transits to state \(j\) after \(k\) time units. Therefore, \(M\) is a \(C(n + 2) \times C(n + 2)\) matrix. The formal derivation...
of \( C(n) \) can be found in [15]:

\[
C(n) = \begin{cases} 
1 & n = 0 \\
\frac{(n+1)(n+3)}{n(n+\frac{3}{4})} & n > 0 \text{ and } n \text{ is odd} \\
\frac{n+\frac{3}{4}}{4} + 1 & n > 0 \text{ and } n \text{ is even}
\end{cases}
\]

Suppose that a wireless link is in state \( i \) at time 0. The probability that the link will become broken at time \( t \) is

\[
P_t(i, t) = \sum_{j \in \text{layer } n+1, n+2} M_{i,j}^t.
\]
The probability that the wireless link is alive at time $t - 1$ but becomes broken at time $t$ is

$$P_2(i, t) = \begin{cases} P_1(i, t) & \text{if } t = 1 \\ P_1(i, t) - P_1(i, t - 1) & \text{if } t > 1 \end{cases}.$$ 

Now consider an $\alpha$-hop route $R = [s_1, s_2, \cdots, s_\alpha]$, where $s_i, i = 1..\alpha$, is the state of the $i$th wireless link in $R$. The probability that $R$ is still alive after $t$ time units is

$$P_3(R, t) = \prod_{i=1}^{\alpha} (1 - P_1(s_i, t)).$$

A path breaks when one or more of its links break. So the probability that $R$ becomes broken after $t$ time units is

$$P_3(R, t) = 1 - P_3(R, t)$$

and the probability that $R$ is alive at time $t - 1$ but becomes broken at time $t$ is

$$P_2(R, t) = \begin{cases} P_3(R, t) & \text{if } t = 1 \\ P_3(R, t) - P_3(R, t - 1) & \text{if } t > 1 \end{cases}.$$ 

Let each wireless LAN card support $m$ rates, $R_0, R_1, \ldots, R_m$, such that rate $R_i, i = 1..m$, will be used if the destination host falls between (including) layers $n_{i-1} + 1$ and $n_i$ from the source, where $n_0 = -1$ and $n_m = n$. For example, reference [2] models an IEEE 802.11b card by $R_1 = 11$, $R_2 = 5.5$, $R_3 = 2$, $R_4 = 1$, $n_0 = -1$, $n_1 = \lfloor \frac{n}{2} \rfloor$, $n_2 = \lfloor \frac{2n}{3} \rfloor$, $n_3 = \lfloor \frac{5n}{6} \rfloor$, and $n_4 = n$. Given the initial state of link $s_i, i = 1..\alpha$, the probability that the link’s rate falling in $R_j$ (i.e., the link’s distance is between layers $n_{j-1} + 1$ and $n_j$) at time $t$ is

$$P_0(s_i, R_j, t) = \sum_{k \in \text{layer}(n_{j-1} + 1) \cdot n_j} M^j_{i,k}.$$ 

Therefore, the bandwidth of $R$ at time $t$ can be modeled by summing the expected transmission rate of the route over all possible rate combination of links in $R$ at time $t$ as follows

$$B(R, t) = \sum_{i_1=1}^{m} \sum_{i_2=1}^{m} \cdots \sum_{i_m=1}^{m} P_0(s_1, R_{i_1}, t) \times P_0(s_2, R_{i_2}, t) \times \cdots \times P_0(s_\alpha, R_{i_\alpha}, t) \times f(R_{i_1}, R_{i_2}, \ldots, R_{i_\alpha}),$$

where the function $f$ is the transmission rate of the route. It will be estimated in next subsection. Finally, the expected throughput of route $R$, denoted by $E(R)$, can be derived by summing the expected route throughput over all possible route lifetime of $R$ as follows

$$E(R) = \sum_{t_1=1}^{\infty} \left( P_3(R, t_1) \times \frac{B(R, t_2)}{t_1} \right).$$

### A. Estimation of the Function $f(\cdot)$

In this subsection, we will propose a method to estimate the throughput of a given $\alpha$-hop route $R = [s_1, s_2, \cdots, s_\alpha]$, where $s_i, i = 1, 2, \ldots, \alpha$, is the state of the $i$th wireless link. Recall that we represent link state as a vector in a 2-dimensional space. So from each $s_i$, we can derive the distance between the two endpoints of the link and the most appropriate rate $r_i$ that should be used by this link. Given such a route $R$, our goal is to derive its transmission rate $f(r_1, r_2, \cdots, r_\alpha)$. An ideal channel condition is assumed in the estimation such that a transmission fails only when collisions occur.

The hosts in routing path $R$ are numbered from 0 to $\alpha$ such that host 0 is the traffic source and host $\alpha$ the sink of the path. Therefore, $s_i$ is the state of the link between host $i - 1$ and host $i$. Except the sink host, we can assign each host $i$ in $R$ an interference group $G_i$, which contains host $i$ and each host $j$ in front of $i$ (i.e., $i > j$) that can sense the signal of $i$ when $i$ is transmitting. Intuitively, hosts in the same group will not transmit at the same time, but hosts in different groups may be allowed to transmit simultaneously. Note that the interference group $G_i$ is defined to make hosts in $G_i$ can not transmit at the same time. If hosts behind host $i$ are included in $G_i$, hosts
Fig. 5. The 9-hop route with its most interfered region including host 5 \sim 9.

in front of host $i$ and hosts behind host $i$ may be allowed to transmit simultaneously. In our estimation, we model the function $f(\cdot)$ by

$$f(r_1, r_2, \cdots, r_\alpha) = \frac{1}{\max_{i=1}^{\alpha-1}\left\{\frac{1}{r_{i+1}} + \sum_{j \in G_i} \frac{1}{r_j}\right\}},$$

where $\max_{i=1}^{\alpha-1}\left\{\frac{1}{r_{i+1}} + \sum_{j \in G_i} \frac{1}{r_j}\right\}$ is the time required to transmit a bit along $R$ in the most interfered region.

The basic concept of our modulation is that host $a$ receiving packet $k + 1$ can not be in the carrier sense range of host $b$ sending packet $k$. In other words, these two packets can be transmitted simultaneously if host $a$ is not in the carrier sense range of host $b$. From the view of pipelining, when packet $k$ arrived at host $b$, packet $k + 1$ has arrived at the host near but out of the carrier sense range of host $a$. Since the slowest stage of a pipeline dominates its throughput, we take the time $T$ of travelling through the most interfered region as the transmission time for packets in $R$. So every packet except the first one in $R$ only takes $T$ to arrive at sink host $\alpha$ after its previous packet arrived at the sink host. Therefore, the expected transmission time for each packet of size $S$ being transmitted in $R$ is

$$T = \max_{i=1}^{\alpha-1}\left\{\frac{1}{r_{i+1}} + \sum_{j \in G_i} \frac{S}{r_j}\right\},$$

and the transmission rate of $R$ is $\frac{S}{T}$. For example, Fig. 5 shows the 9-hop route with its most interfered region including host 5 \sim 9. The links that can transmit concurrently are indicated by dashes. We can find that when packet $k$ arrived at sink host 9, packet $k + 1$ has arrived at host 4. It is because that when host 3 is sending packet $k + 1$ to host 4, host 8 can send packet $k$ to sink host 9 at the same time without interfering the receiving of host 4. Therefore, packet $k + 1$ in the 9-hop route only takes the time of travelling through hosts 5 \sim 8 to arrive at sink host 9 after packet $k$ arrived at the sink host. Accordingly, the transmission rate of the 9-hop route is $\frac{1}{\sum_{j \in C_9} \frac{1}{r_j}} = \frac{1}{\frac{1}{r_5} + \frac{1}{r_6} + \frac{1}{r_7} + \frac{1}{r_8} + \frac{1}{r_9}}$.

IV. SIMULATION RESULTS

In this section, we present our simulation results. Most current products of IEEE 802.11b have a transmission distance of 150 \sim 300 meters. We set the radius of each hexagonal cell to 10 meters, so hosts’ transmission range is around $n = 15 \sim 30$ layers. The carrier sense range is set to be the same with the transmission range, and the mobile host is set to randomly select its roaming direction per time unit. Each time unit is set to 10 seconds, so as to model the roaming speed of pedestrians (around 1 m/s). The saturated traffic and unlimited buffer are used in our simulation, and the roaming speed of each mobile host is set to 1 m/s.

First, we try to determine the level of accuracy. Observe that index $t_1$ in Eq. (1) ranges from 1 to infinity. This is computationally infeasible. So we need to determine an upper bound for $t_1$ (called $t_{1,max}$ below). We randomly generate five routing paths with 1, 3, 6, 9, and 12 hops, respectively. We calculate their expected throughput by
varying $t_{\text{max}}$ from 100 to 1000. The results are in Fig. 6 for $n = 15$ and 25, respectively. Since $E(R)$ stabilizes at $t_{\text{max}} = 300$, we will set $t_{\text{max}} = 1000$ in the rest of the simulations.

Our results can be used to help route selection in a MANET. Hop count is probably the most widely used route selection criterion. Our result may provide an alternative choice if throughput is the main concern, especially under a multi-rate environment. We pick a source cell and a destination cell, and place some relay hosts between them which are separated uniformly. We evaluate the expected route throughput by varying the number of relay hosts (and thus path length that is the number of links in the path). Fig. 7 shows our results for $n = 15$ and 25, respectively.

In both cases, we see that the throughput increases with the path length initially, but decreases afterwards after certain thresholds. In fact, there are two contradicting factors here. A very small path length implies a low transmission rate in each hop, thus leading to low path throughput. On the contrary, a longer path implies potentially higher rates and the higher degree of spectral reuse, but may risk a higher probability of existence of low-rate links in the path (thus becoming a bottleneck). Our result may be used here to make a smart choice.

Fig. 7 also contains comparisons of simulation and analytical results. In each simulation, we evaluate the throughput of the path every time unit until it is broken and then calculate the average throughput. Each simulation is repeated 20,000 times to capture the random roaming of mobile hosts, and then we take the average throughput. As can be seen, the simulated and analytical results are quite close, which justifies the correctness of our derivation.

V. Conclusions

In this paper, we have shown how to formulate the throughput given a path in which hosts roam around in a random walk model and the communication interfaces have the rate adaptive capability. As far as we know, this issue has not been carefully studied yet. Simulation results show that the proposed formulation can be used to evaluate path throughput accurately. We believe that the path throughput is a better metric than the traditional metrics, such as the hop count, for route selection in multi-rate ad hoc networks and that the proposed mechanism can be easily embedded into most of the current routing protocols for mobile ad hoc networks.

References

Fig. 6. Expected route throughput vs. $t^\text{max}_1$: (a) $n = 15$ and (b) $n = 25$. 

R1: [3,3] 
R2: [2,2] <6,0|7,2> 
R3: [3,2] <6,2|8,3> <2,0|6,4> <2,2> 
R4: [5,2] <7,3|8,5> <0,0|8,1> <10,0|5,0|8,2> 
R5: [8,1|6,1|5,2|0|9,3|7,3|8,6|3,1|7,1|7,0|6,0|6,5> 

(a) 

R1: [12,5] 
R2: [10,8|12,2|5,3] 
R3: [9,7|6,4|13,2|2,0|8,4|5,2] 
R4: [7,1|15,3|6,4|3,2|4,1|16,2|10,3|12,1|7,5] 
R5: [19,2|6,2|10,7|11,2|21,0|2,1|11,4|7,0|4,2|8,3|21,1|16,3] 

(b)
Fig. 7. Expected route throughput vs. path length: (a) $n = 15$ and (b) $n = 25$. 
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