Design of 2-D Digital Filters with Almost Quadrantal Symmetric Magnitude Response without 1-D Separable Denominator Factor Constraint

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***Abstract*— A design approach is presented for 2-D digital filters possessing approximate quadrantal magnitude symmetry without the constraint of the denominator having only 1-D separable factors. To ensure the BIBO stability of the filter, the planar least square inverse stabilization approach is employed. It is illustrated through design examples that the proposed approach results in filters with sharper transition band and lower error relative to the given filter specifications. Also, for certain cases, it is shown that a lower order non-separable denominator design can achieve the same result as a higher order separable denominator design, thus providing savings in the number of multipliers. Finally, 2-D VLSI realizations without global broadcast are presented for the optimized transfer function with non-separable denominator factors and approximate quadrantal symmetry.**

I. INTRODUCTION

In the design of 2-D digital filters possessing exact magnitude symmetry, the transfer function must have a denominator having only 1-D (stable) separable factors. This is needed to ensure that the 2-D filter is BIBO stable while possessing the said symmetry. As the denominator is in a constrained form it may not be possible to meet certain specifications such as sharp cut off in the transition band of the filter magnitude specs. This can force the use of higher order transfer function in the separable case resulting in more expensive implementation. In this paper we initiate a filter design procedure with 2-D non separable denominator factors in the filter transfer function to achieve an almost quadrantal symmetric magnitude response. The numerator of the filter transfer function is still chosen as a quadrantal symmetric polynomial allowing us to reduce the number of multipliers in the final realization as well as to attain the near quadrantal symmetric overall response. The example shown is a Fan filter design with various stopband angles. The optimization results show sharper cut off transition band response with non-separable denominator when compared with exact quadrantal symmetric response using a transfer function with separable denominator. The obtained response also shows that the error in the optimization to approximate the specs is much smaller in the non-separable case while closely traversing the quadrantal symmetric magnitude response contours. The stability problem in the non-separable case that is discussed is solved by using the planar least squares inverse (PLSI) stabilization approach first established by Anderson and Jury [9,10]. Finally, a 2-D VLSI realization without global broadcast is presented for the optimized transfer function with non-separable denominator factors.

II. PRELIMINARIES

A general 2-D IIR transfer function can be represented as in (1), where N_1xN_2 is the order of the filter. Without loss of generality, we will assume $N_1=N_2=N$ in discussing the filters.

$$H(z_1,z_2)=\frac{N(z_1,z_2)}{D(z_1,z_2)}=\frac{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}a_{ij}z_1^{\,i}z_2^{\,j}}{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}b_{ij}z_1^{\,i}z_2^{\,j}} \tag{1}$$

In most 2-D filter applications, the filter transfer function possesses some form of symmetry in its magnitude response. There are many types of symmetries [5]. Here, we will focus on one of the symmetries, namely, quadrantal symmetry. A filter transfer function possesses quadrantal symmetry in its magnitude response if:

$$\left|H\left(e^{j\theta_1},e^{j\theta_2}\right)\right|=\left|H\left(e^{-j\theta_1},e^{j\theta_2}\right)\right|=\left|H\left(e^{j\theta_1},e^{-j\theta_2}\right)\right|=\left|H\left(e^{-j\theta_1},e^{-j\theta_2}\right)\right|$$

The presence of symmetry in the 2-D frequency response induces certain relationship among the filter coefficients which can result in fewer multipliers in the implementation.

It has been shown [5] that quadrantal symmetry implies that the transfer function denominator has to be 1-D product separable, i.e. $D(z_1,z_2)=D_1(z_1)\cdot D_2(z_2)$ and the numerator coefficients need to have the relationship: $a_{ij}=a_{(N-i)j}$ or $a_{ij}=a_{i(N-j)}$ for all *i, j*. However, this denominator separability constraint can be over restrictive and makes it difficult to design filters with sharp cutoff in the transition band [8]. In this paper, we show a filer design procedure utilizing the same quadrantal symmetry numerator but with the denominator being a 2-D non-separable polynomial. This is shown to yield fan filter designs with sharp cutoff. The final filter magnitude response possesses approximate rather than exact quadrantal symmetry, but is sufficient for most applications.

One issue with using a non-separable denominator is ensuring its stability. The approach we use here is to construct the denominator using 1x1 2-D factors, i.e.

$$D(z_1,z_2)=\prod_{i=1}^{N}\left(d_{0i}+d_{1i}\cdot z_1+d_{2i}\cdot z_2+d_{3i}\cdot z_1\cdot z_2\right) \tag{2}$$

Any unstable 1x1 factor can then be stabilized through PLSI polynomial approach. The denominator factors are then multiplied out to yield the general polynomial form in (1) for realization.

III. PLANAR LEAST SQUARE INVERSE POLYNOMIAL STABILIZATION

Let the 2-D polynomial $A(z_1,z_2) = a_{00} + a_{10} \cdot z_1 + a_{01} \cdot z_2 + a_{11} \cdot z_1 \cdot z_2$ be the polynomial to be stabilized and let $B(z_1,z_2) = b_{00} + b_{10} \cdot z_1 + b_{01} \cdot z_2 + b_{11} \cdot z_1 \cdot z_2$ be the planar least square inverse (PLSI) polynomial of $A(z_1,z_2)$. Following the steps in the reference [9,10], it can be shown that $B(z_1,z_2)$ which is an approximate inverse of $A(z_1,z_2)$ is a bounded input and bounded output (BIBO) stable polynomial, independent of $A(z_1,z_2)$ polynomial's stability. This means $B(z_1,z_2)$ is devoid of zeros in the closed unit bi-disk. Now let us take another PLSI of $B(z_1,z_2)$ and call it $C(z_1,z_2)$. The polynomial $C(z_1,z_2)$ is an approximate inverse of $B(z_1,z_2)$ and is also a BIBO stable polynomial by the same arguments. It is to be noted that $C(z_1,z_2)$ now becomes an approximate equivalent of the starting polynomial $A(z_1,z_2)$ in the least squares minimization sense and thus possesses the approximate magnitude response of $A(z_1,z_2)$.

In conclusion, $C(z_1,z_2)$ which is the double PLSI of $A(z_1,z_2)$, is a BIBO stable polynomial independent of whether $A(z_1,z_2)$ is a stable polynomial or not. Further $C(z_1,z_2)$ has the approximate magnitude response of $A(z_1,z_2)$. So, after the optimization process, if the resulting denominator factor $A(z_1,z_2)$ of the transfer function is an unstable polynomial, then, it could be replaced by its double PLSI polynomial $C(z_1,z_2)$. The generalization of this result for any order $A(z_1,z_2)$ is still an open research problem.

IV. DESIGN OF FAN FILTER

Optimization is used to obtain the transfer function that satisfies the fan filter magnitude response specifications given in Figure 1. The filter stopband has an angle of 2□. The filter has a narrow transition band, specified by x_1=0.0157 and x_2=0.113.

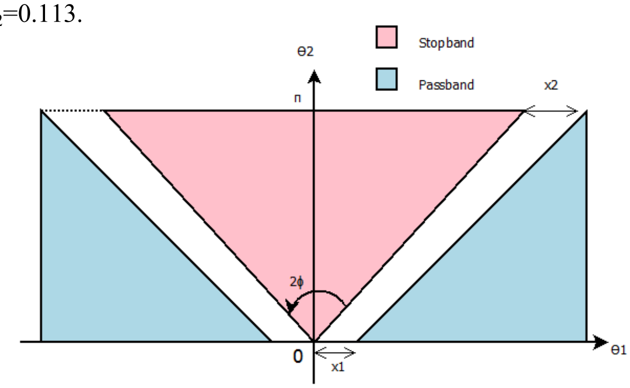

Figure 1. Fan filter specifications

For the optimization, the numerator of the transfer function is assumed to be quadrantal symmetric, i.e. $a_{ij} = a_{(N-i)j}$, and the denominator is of the form in (2). After optimization, the denominator factors are stabilized, if necessary, using PLSI approach and multiplied out to become a regular polynomial for implementation.

The optimization objective is to minimize the least squared error between the filter magnitude response and the give filter specifications. The initial values to start the optimization are obtained using genetic algorithm. The final results are shown in Table 1 for a filter order of 4x4 and different stopband angles. It can be seen that the objective function error is always smaller for the non-separable denominator design compared to the traditional 1-D separable denominator design.

TABLE I. COMPARISON OF 4X4 FILTERS

Filter order	Filter stopband angle (ϕ)	Non-Sep deno error	Sep deno Error
4x4	15°	23.69	32.27
4x4	25°	16.15	27.75
4x4	35°	9.77	32.16
4x4	45°	14.72	28.63

The magnitude contour plot for the non-separable denominator design with filter stopband of 35 degree is shown in Figure 2. The corresponding contour plot for the separable denominator design in shown in Figure 3. It can be seen that non-separable denominator design has a much sharper transition band. It also displayed very impressive quadrantal symmetry despite it being not exact.

The contour plots for a stopband angle of 45 degrees are shown in Figures 4 and 5. Again it can be seen that the non-separable denominator design has a sharper transition band. The quadrantal symmetry in the plot for the non-separable design, although not exact, is still quite acceptable.

To further illustrate the advantage, 2x2 filters are obtained for the non-separable denominator design with stopbands of 35 and 45 degrees. The results are shown in Table 2. The contour plots are shown in Figures 6 & 7.

TABLE II. 2X2 FILTERS RESULTS

Filter order	Filter stopband angle (ϕ)	Non-Sep deno error
2x2	35°	24.62
2x2	45°	20.61

It can be observed that the 2x2 non-separable denominator design can achieve similar error as the 4x4 separable denominator design. This will result in multiplier savings in the filter realization as the 2x2 non-separable denominator design requires only 14 multipliers while the 4x4 separable denominator design requires 23 multipliers.

V. VLSI Filter Structures without Global Broadcast

A VLSI filter structure realizing a quadrantal symmetric numerator and non-separable denominator is shown in Figure 8. It can be used to realize the optimized transfer function just obtained. Because of space constraints, only an order 2x2 structure is shown; this can easily be generalized to higher orders. An alternate structure based on its transpose is shown in Figure 9.

Here, we assume that the filter is used to process a square image of size *M*x*M* and the pixel values in the image are fed to the filter in raster-scan mode, i.e. the input sequence is *x(0,0)*, *x(0,1)*, …, *x(0,M-1)*, *x(1,0)*, *x(1,1)*, … etc. So we replace z_2^{-1} of the transfer function by a single delay register, z^{-1}, and z_1^{-1} by a shift register of length M, z^{-M}, provided *M>N*.

Note that the special arrangement of the delays in the structures is to eliminate global broadcast of the signals and also to control the critical period.

VI. Conclusion

In this paper it has been shown that one can obtain almost quadrantal symmetric frequency magnitude response by using a non-separable 2-D polynomial factor of degree (1x1) in the denominator of the filter transfer function. The numerator of the transfer function chosen is still a polynomial with quadrantal symmetric properties. It has also been shown that error in the optimization is small and the rate of fall off in the magnitude response in the transition band is steeper when compared with the separable denominator transfer function. This may be due to the fact that there are extra parameters to optimize and also by the nature of the non-separable 2-D denominator polynomial itself. Also, the BIBO stability is always guaranteed by employing the planar least square inverse stabilization approach for degree (1x1) polynomial, first established by Anderson and Jury [9,10]. It is also shown that, for certain cases, the lower degree transfer function with non-separable denominator could be used to satisfy the given specification than with transfer function with 1-D separable denominator factors. A VLSI implementation of the filter is given in the end. In future studies we will explore the design with other types of symmetries and also the power consumption of separable and non-separable denominator structures. Also, in future, more filter design examples with different specifications need to be studied for non-separable case to arrive at the pattern of the advantages studied in this paper.

Acknowledgement

Hari C. Reddy and I-Hung Khoo would like to express their appreciation to Dr. P. K. Rajan of Tennessee Tech for his discussions on the significance of the design approach presented in this work.

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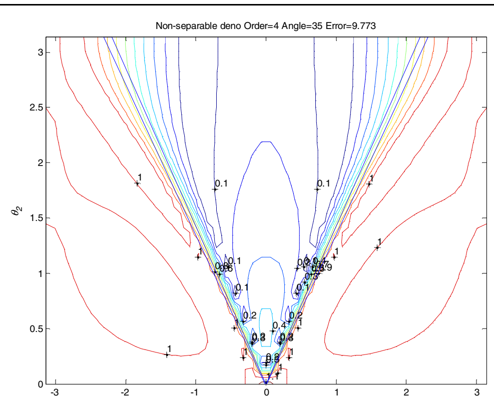

Figure 2. 4x4 non-separable denominator filter (stopband angle=35°)

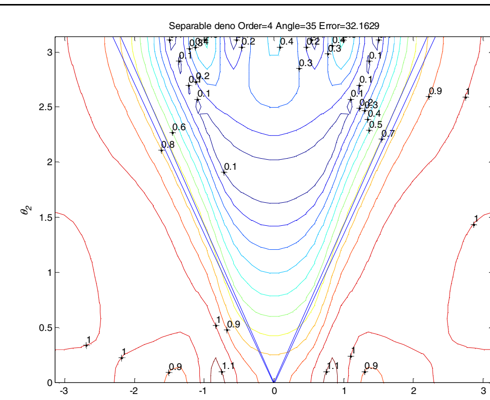

Figure 3. 4x4 separable denominator filter (stopband angle=35°)

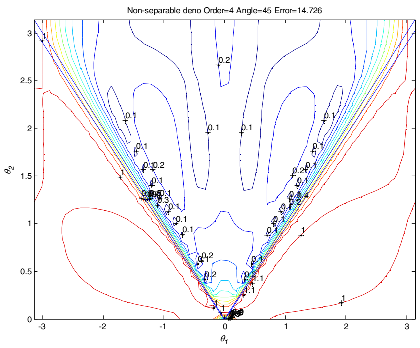

Figure 4. 4x4 non-separable denominator filter (stopband angle=45°)

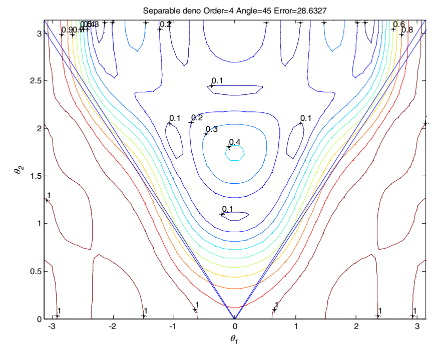

Figure 5. 4x4 separable denominator filter (stopband angle=45°)

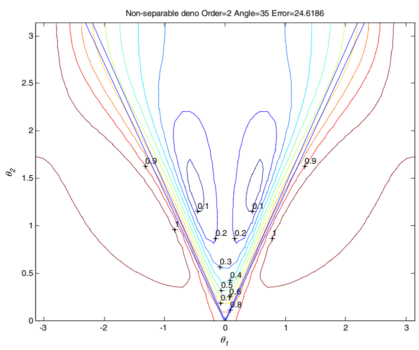

Figure 6. 2x2 non-separable denominator filter (stopband angle=35°)

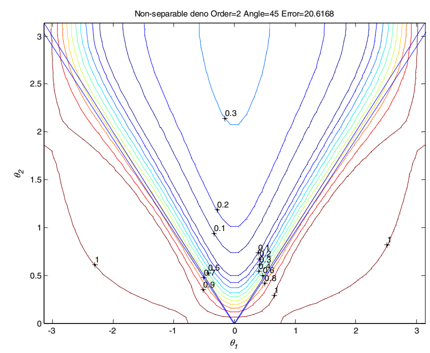

Figure 7. 2x2 non-separable denominator filter (stopband angle=45°)

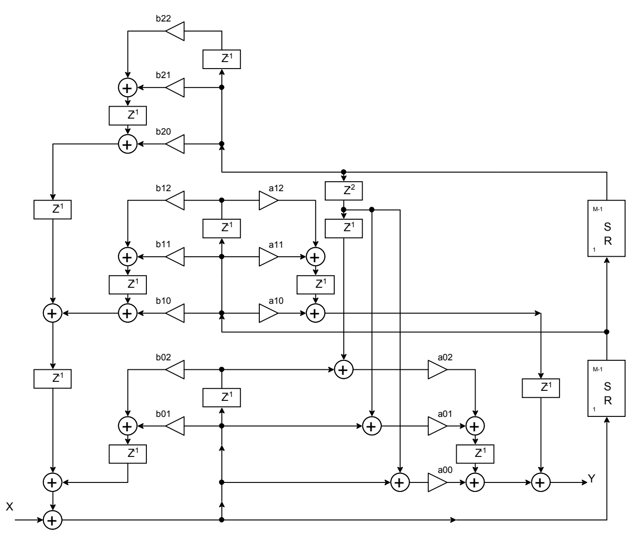

Figure 8. 2x2 filter structure with quadrantal symmetric numerator and non-separable denominator (Type I).

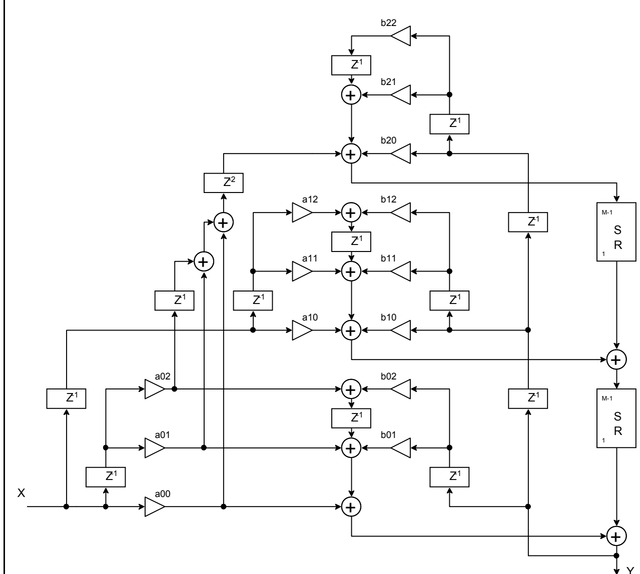

Figure 9. 2x2 filter structure with quadrandral symmetric numerator and non-separable denominator (Type 2)