Delta Operator Based 2-D VLSI Filter Structures without Global Broadcast and Incorporation of the Quadrantal Symmetry

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***Abstract*— Having local data communication (without global broadcast of signals) among the elements is important in VLSI designs. Recently, 2-D systolic digital filter architectures were presented which eliminated the global broadcast of the input and output signals. The delta discrete-time operator based 1-D and 2-D digital filters (in γ-domain) were shown to offer better numerical accuracy and lower coefficient sensitivity in narrow-band filter designs when compared to the traditional shift-operator formulation. Further, the complexity in the design and implementation of 2-D filters can be reduced considerably if the symmetries that might be present in the frequency responses of these filters are utilized. With this motivation we present new 2-D VLSI filter structures, without global broadcast, using delta discrete-time operator for the first time. We also present frame works in γ-domain that realizes 2-D filters possessing quadrantal symmetry in its magnitude response. The separable denominator and quadrantal symmetry structures have the advantage of reduced number of multipliers while ensuring the 2-D filter stability.**

I. INTRODUCTION

Two-dimensional (2-D) digital filters find applications in many digital signal processing areas such as image processing, beamforming, and seismic data processing. Although 2-D digital filters can be simulated on a general purpose computer, for applications involving high data rate, such as real time image processing, dedicated computing structures are needed in order to meet the high throughput demands. Networks using structures such as systolic arrays are popular candidates for VLSI ASIC implementation due to the regularity and modularity of the processing elements involved. Having local data communication (without global broadcast of signals) among the elements is important in such VLSI designs. In [1,2], 2-D systolic digital filter architectures were presented which eliminated the global broadcast of the input and output signals in previous architectures [3,4]. In addition, in [2], new structures realizing transfer functions with separable denominators and having diagonal magnitude symmetry were presented. It is well know that symmetry in the filter response can be used to reduce the number of multipliers in the filter realization. Recently in [5], eight symmetry filter structures were presented. This creates the motivation for this work to develop a generalized formulation for new 2-D delta operator based filter structures with symmetry, following the results in [6]. The delta discrete-time operator was introduced by Middleton and Goodwin in 1990 [7]. By replacing the conventional shift operator (q) in the z-domain approach with the delta discrete-time operator (δ), one can overcome the numerical ill-conditioning and coefficient sensitivity problems faced by the conventional z-domain filters when the filter poles are clustered near z=1.

We start by discussing the nature of 2-D filter transfer functions and symmetry for delta operator formulations in Sections 2 and 3. Then in Section 4, the various 1-D sub-blocks used in the 2-D filter structures are presented. Here, a general digital two-pair approach is used to describe the sub-blocks which consist of direct-form polynomial filter in one of the frequency variables. Then, by applying the sub-blocks in various frameworks, 2-D structures realizing different transfer functions are obtained. These include general IIR transfer function (Section 5), separable denominator transfer function (Section 6), and quadrantal symmetry transfer function (Section 7). After this, the multipliers required for the filter structures are discussed together with the savings.

II. PRELIMINARIES

Since the introduction by Middleton and Goodwin, delta operator based designs have been studied extensively in the area of digital control systems and signal processing due to their excellent finite wordlength performance under fast sampling [8]. The delta operator is defined as:

$$\delta\left[x(nT)\right]=\frac{x(nT+T)-x(nT)}{T} \tag{1}$$

where T may denote the sampling period or a constant.

It is easy to see that the relationship between the delta operator and the shift operator is given by δ=(q-1)/T. In the transform domain, δ is represented by the transform variable γ= (z-1)/T. Or, as a causal element: $\gamma^{-1}=T\cdot z^{-1}/\left(1-z^{-1}\right)$. Following the notations in [9], let $\gamma_i=(z_i-1)/T_i$ for i = 1, 2 represent the delta operator in the transform domain for 2-D systems. Then the transfer functions of a 2-D system $H(z_1,z_2)$ in the z-domain and $H_\gamma(\gamma_1,\gamma_2)$ in the γ-domain are related as follows:

$$H_\gamma(\gamma_1,\gamma_2)=H(z_1,z_2)\big|_{z_i=(1+T_i\gamma_i)}\quad,\ i=1,2 \tag{2}$$

where $$H_\gamma(\gamma_1,\gamma_2)=\frac{Y(\gamma_1,\gamma_2)}{X(\gamma_1,\gamma_2)}=\frac{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}c_{ij}\gamma_1^{-i}\gamma_2^{-j}}{1-\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}d_{ij}\gamma_1^{-i}\gamma_2^{-j}} \tag{3}$$

III. 2-D SYMMETRY

Symmetry present in the frequency response induces a relation among the filter coefficients which reduces the number of multipliers in an implementation structure [9]. There are many possible types of symmetries in the magnitude response such as quadrantal, diagonal, rotational, octagonal symmetries etc. In this paper, we will focus on quadrantal symmetry.

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If $P(\gamma_1,\gamma_2)$ is a 2-D γ-domain polynomial, then its frequency response is given by $P\left(\frac{e^{j\theta_1}-1}{T},\frac{e^{j\theta_2}-1}{T}\right)$. The magnitude squared function of the frequency response is given by:

$$\begin{aligned} F(\theta_1,\theta_2) &= P\left(\frac{e^{j\theta_1}-1}{T},\frac{e^{j\theta_2}-1}{T}\right)\cdot P\left(\frac{e^{-j\theta_1}-1}{T},\frac{e^{-j\theta_2}-1}{T}\right) \\ &= P(\gamma_1,\gamma_2)\cdot P\left(\frac{-\gamma_1}{1+T\gamma_1},\frac{-\gamma_2}{1+T\gamma_2}\right)\Bigg|_{\gamma_i=\frac{e^{j\theta_i}-1}{T}},\ \text{i=1,2} \end{aligned} \tag{4}$$

If the magnitude squared function possesses quadrantal symmetry, then

$$F(\theta_1,\theta_2) = F(-\theta_1,\theta_2) = F(\theta_1,-\theta_2) = F(-\theta_1,-\theta_2) \quad ,\forall\ (\theta_1,\theta_2) \tag{5}$$

Expressing (5) in terms of the polynomial yields:

$$P(\gamma_1,\gamma_2)\cdot P\left(\frac{-\gamma_1}{1+T\gamma_1},\frac{-\gamma_2}{1+T\gamma_2}\right) = P\left(\frac{-\gamma_1}{1+T\gamma_1},\gamma_2\right)\cdot P\left(\gamma_1,\frac{-\gamma_2}{1+T\gamma_2}\right) \tag{6}$$

Now, applying the unique factorization property of 2-variable polynomials to (6), it can be seen that $P(\gamma_1,\gamma_2)$ should satisfy one of the following two conditions:

(i) $$P(\gamma_1,\gamma_2) = k_1\cdot P\left(\frac{-\gamma_1}{1+T\gamma_1},\gamma_2\right) \tag{7}$$

(ii) $$P(\gamma_1,\gamma_2) = k_2\cdot P\left(\gamma_1,\frac{-\gamma_2}{1+T\gamma_2}\right) \tag{8}$$

where k_1 and k_2 are real constants.

To derive the symmetry constraint on the polynomial, we first observe that the term $\gamma_i^{-2}+T\gamma_i^{-1}$ is self inverse in γ_i, i.e.:

$$\gamma_i^{-2}+T\gamma_i^{-1}\Big|_{\gamma_i=\frac{-\gamma_i}{1+T\gamma_i}} = \gamma_i^{-2}+T\gamma_i^{-1} \tag{9}$$

Therefore, polynomials of the form $P\left(\gamma_1^{-2}+T\gamma_1^{-1},\gamma_2\right)$ and $P\left(\gamma_1,\gamma_2^{-2}+T\gamma_2^{-1}\right)$ satisfies (7) and (8) respectively. This means that polynomials that can be expressed in terms of $\gamma_1^{-2}+T\gamma_1^{-1}$ or $\gamma_2^{-2}+T\gamma_2^{-1}$ will possess quadrantal symmetry. These polynomial forms will be used in the numerator of the delta operator IIR filter transfer function with quadrantal symmetry. They cannot be used for the denominator due to stability problem. Instead, the denominator is chosen as separable, i.e. $P_1(\gamma_1)\cdot P_2(\gamma_2)$. It is easy to see that $P_1(\gamma_1)$ and $P_2(\gamma_2)$ satisfies (8) and (7) respectively, so their product possesses quadrantal symmetry. The advantage of a separable denominator is that it is easy to ensure the stability. The symmetry constraints will be used to obtain filter structures with fewer number of multipliers.

IV. TWO PAIR 1-D FILTER SUB-BLOCKS

The transfer function in (3) can also be expressed as:

$$H_\gamma(\gamma_1,\gamma_2) = \frac{Y(\gamma_1,\gamma_2)}{X(\gamma_1,\gamma_2)} = \frac{\sum_{i=0}^{N_1} E_i^g(\gamma_2)\cdot\gamma_1^{-i}}{1-\sum_{i=0}^{N_1} D_i^g(\gamma_2)\cdot\gamma_1^{-i}} \tag{10}$$

where $E_i^g(\gamma_2)=\sum_{j=0}^{N_2} c_{ij}\gamma_2^{-j}$ and $D_i^g(\gamma_2)=\sum_{j=0}^{N_2} d_{ij}\gamma_2^{-j}$ are 1-D polynomials in γ_2 to be realized by the sub-blocks discuss here. These sub-blocks will be used in the filter frameworks in Sections 5,6,7 to realize various 2-D transfer functions.

In our discussion, we assume that the filter is used to process a square image of size *M*x*M* and the pixel values in the image are fed to the filter in raster-scan mode, i.e. the input sequence is *x(0,0)*, *x(0,1)*, …, *x(0,M-1)*, *x(1,0)*, *x(1,1)*, … etc. Thus, z_2^{-1} can be represented by a single delay register, z^{-1}, and z_1^{-1} by a shift register of length M, z^{-M}, provided $M>N_2$. In terms of the delta operator, this implies replacing $\gamma_2^{-1}=T\cdot z_2^{-1}/\left(1-z_2^{-1}\right)$ with $\gamma^{-1}=T\cdot z^{-1}/\left(1-z^{-1}\right)$, and $\gamma_1^{-1}=T\cdot z_1^{-1}/\left(1-z_1^{-1}\right)$ with a delta operator shift register: $T\cdot z^{-M}/\left(1-z^{-M}\right)$. Without loss of generality, we will assume $N_1=N_2=N$ and T=1 in discussing the filters.

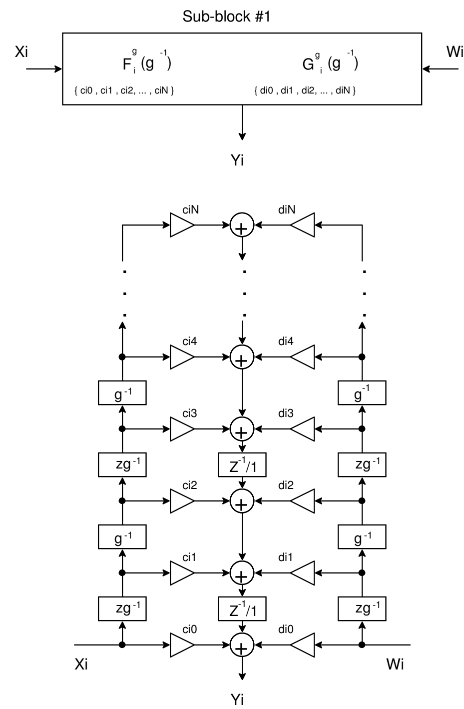

Figure 1. Sub-block #1 (2-inputs-1-output)

The sub-blocks #1 and #2 for use in the delta-operator realization are shown in Fig. 1 and 2 respectively. In the diagrams, the g^{-1} and zg^{-1} elements are equivalent to $\gamma^{-1}=\frac{z^{-1}}{1-z^{-1}}$ and $z.\gamma^{-1}=z\cdot\frac{z^{-1}}{1-z^{-1}}=\frac{1}{1-z^{-1}}$ respectively. They can be implemented as shown in Fig. 3 and 4. Note that the cascade connection of the g^{-1} and zg^{-1} elements yields the self inverse term $\left(\gamma^{-2}+\gamma^{-1}\right)$ needed for quadrantal symmetry. The $z^{-1}/1$ element can be configured to either implement a delay or a passthrough.

Sub-block #1 has 2 inputs and 1 output. With the $z^{-1}/1$ element configured as a delay, it realizes the following two polynomial functions:

$$F_i^g\left(\gamma^{-1}\right)=\frac{Y_i}{X_i}\Big|_{W_i=0}=\sum_{j=0}^{N}c_{ij}\gamma^{-j}\ ,\quad G_i^g\left(\gamma^{-1}\right)=\frac{Y_i}{W_i}\Big|_{X_i=0}=\sum_{j=0}^{N}d_{ij}\gamma^{-j} \tag{11}$$

With the $z^{-1}/1$ element configured as a passthrough and with $c_{ij}=0$, $d_{ij}=0$ for j=1,3,5…, it realizes the following functions, which can be used for quadrantal symmetry.

$$\begin{aligned} F_i^g\left(\gamma^{-1}\right) &= \left.\frac{Y_i}{X_i}\right|_{W_i=0} = c_{i0} + c_{i2}\left(z\cdot\gamma^{-2}\right) + c_{i4}\left(z\cdot\gamma^{-2}\right)^2 + \ldots\ldots \\ &= \sum_{j=0,2,4\ldots}^{N} c_{ij}\left(z\cdot\gamma^{-2}\right)^{\frac{j}{2}} = \sum_{j=0,2,4\ldots}^{N} c_{ij}\left(\gamma^{-2}+\gamma^{-1}\right)^{\frac{j}{2}} \\ G_i^g\left(\gamma^{-1}\right) &= \left.\frac{Y_i}{W_i}\right|_{X_i=0} = \sum_{j=0,2,4\ldots}^{N} d_{ij}\left(\gamma^{-2}+\gamma^{-1}\right)^{\frac{j}{2}} \end{aligned} \tag{12}$$

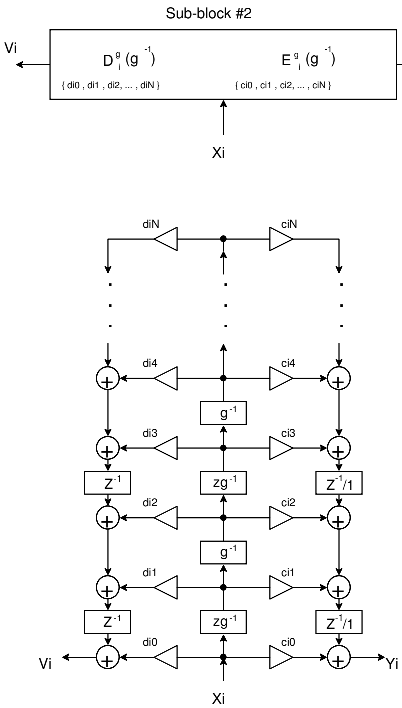

Figure 2. Sub-block #2 (1-input-2-outputs)

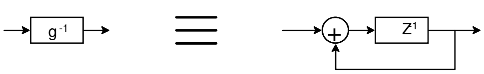

Figure 3. Realization of γ^{-1}

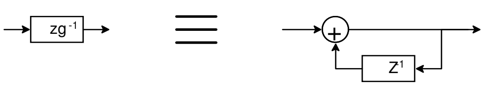

Figure 4. Realization of $z.\gamma^{-1}$

The other delta operator sub-block is Sub-block #2 which has 1 input and 2 outputs. It realizes the following polynomial functions with the $z^{-1}/1$ element configured as a delay:

$$D_i^g\left(\gamma^{-1}\right) = \frac{V_i}{X_i} = \sum_{j=0}^{N} d_{ij}\gamma^{-j} \quad , \quad E_i^g\left(\gamma^{-1}\right) = \frac{Y_i}{X_i} = \sum_{j=0}^{N} c_{ij}\gamma^{-j} \tag{13}$$

With the $z^{-1}/1$ element configured as a passthrough and with $c_{ij}=0$ for j=1,3,5…, it realizes the following different E_i^g function, which can be used for quadrantal symmetry:

$$E_i^g\left(\gamma^{-1}\right) = \frac{Y_i}{X_i} = \sum_{j=0,2,4\ldots}^{N} c_{ij}\left(\gamma^{-2}+\gamma^{-1}\right)^{\frac{j}{2}} \tag{14}$$

V. Filter Framework for Realizing General Transfer Function

Sub-block #2 discussed in the previous section is used in the filter Framework A in Fig. 5 to realize the 2-D delta operator transfer function in (10). Note the additional delays added at the input and output branches are to eliminate the global broadcast of the input and output signals. The $Z^{-1}/1$ elements in sub-block#2 are configured as delays. Note that the delta operator shift register (gSR) is implemented as shown in Fig 6 which realizes $z\cdot\gamma_1^{-1}$. It consists of a regular shift register with a loop and delay around it. It is easy to verify using Mason's gain formula that the structure possesses the transfer function in (10). Recall that $\gamma^{-1}=\gamma_2^{-1}$.

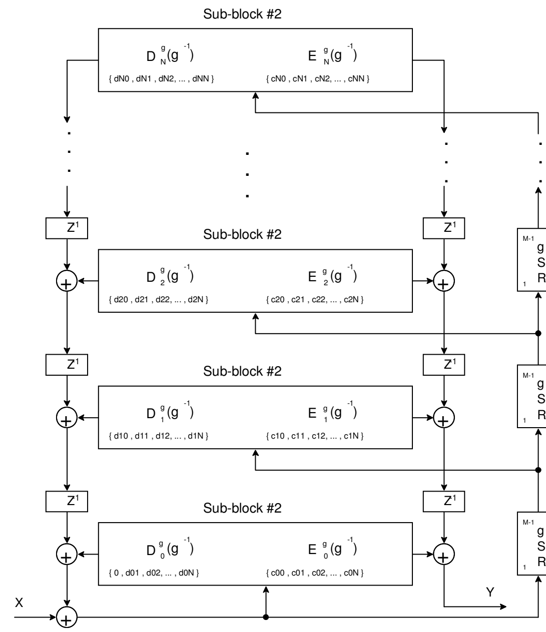

Figure 5. Filter framework A

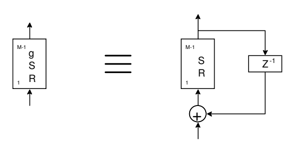

Figure 6. Implementation of delta operator shift register

VI. Separable Denominator Filter Framework

By mixing the sub-blocks in specific ways, filter framework realizing transfer functions with separable denominator of the form in (15) can be obtained. The idea is to form two non-touching loops in different variables as per Mason's gain formula.

$$\begin{aligned} \frac{Y}{X} &= \frac{\sum_{i=0}^{N}\sum_{j=0}^{N} c_{ij}\gamma_1^{-i}\gamma_2^{-j}}{\left(1-\sum_{i=1}^{N} d_{i0}\gamma_1^{-i}\right)\cdot\left(1-\sum_{j=1}^{N} d_{0j}\gamma_2^{-j}\right)} \\ &= \frac{E_0^g\left(\gamma_2\right)+\sum_{i=1}^{N} F_i^g\left(\gamma_2\right)\cdot\gamma_1^{-i}}{\left[1-D_0^g\left(\gamma_2\right)\right]\cdot\left[1-\sum_{i=1}^{N} G_i^g\cdot\gamma_1^{-i}\right]} \end{aligned} \tag{15}$$

The separable denominator transfer function has several advantages over the general one in (3). Firstly, the stability can be checked by simply solving for the poles of the two 1-D polynomials, and any unstable pole is easy to stabilize. Secondly, the separable denominator requires fewer multipliers to realize. Thirdly, the separable denominator is required in realizing stable magnitude responses possessing various symmetries (except for the diagonal symmetry) [9].

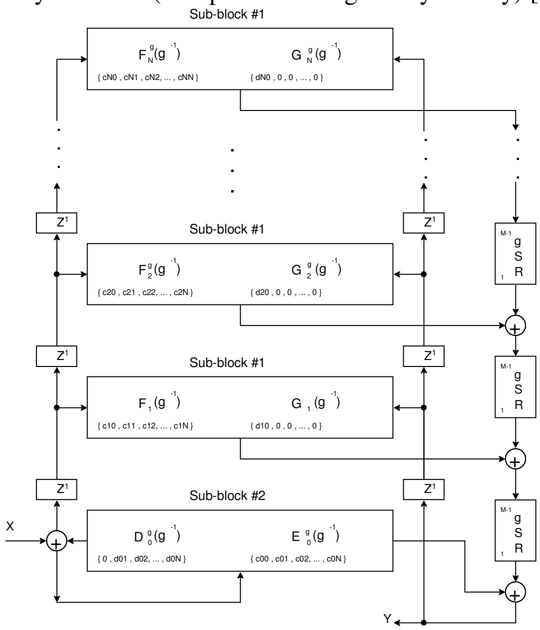

Figure 7. Filter framework B (separable denominator)

Filter Framework B (Fig 7) is used to realize the delta operator separable denominator transfer function. It uses sub-block #2 at the bottom while the rest are sub-block #1. The $Z^{-1}/1$ elements in sub-blocks #1 and #2 are configured as delays.

VII. QUADRANTAL SYMMETRY FILTER FRAMEWORK

The delta operator filter structure with quadrantal symmetry is shown in Fig 8. It realizes the following transfer function:

$$\frac{Y}{X}=\frac{\sum_{i=0}^{N}\sum_{j=0,2,4...}^{N} c_{ij}\gamma_1^{-i}\left(\gamma_2^{-2}+\gamma_2^{-1}\right)^{\frac{j}{2}}}{\left(1-\sum_{i=1}^{N} d_{i0}\gamma_1^{-i}\right)\cdot\left(1-\sum_{j=1}^{N} d_{0j}\gamma_2^{-j}\right)} \quad (16)$$

It is based on Framework B. Note that the $Z^{-1}/1$ element in sub-blocks #1 and #2 need to be configured as passthrough. The structure can only realize even order transfer function. The changes to the sub-block parameters are highlighted in red in the figure. Note that although the structure shown is 2x2, it can easily be generalized to higher orders.

VIII. COMPARISON OF MULTIPLIERS REQUIRED

The quadrantal symmetry structure has the lowest number of multipliers compared to all the other structures. The filter framework realizing regular 2-D IIR transfer function requires $2(N+1)^2-1$ multipliers. The separable denominator framework requires fewer multipliers: $(N+1)^2+2N$. The quadrantal symmetry structure requires the least number of multipliers: only $(N/2+1)\cdot(N+1)+2N$ where N is even.

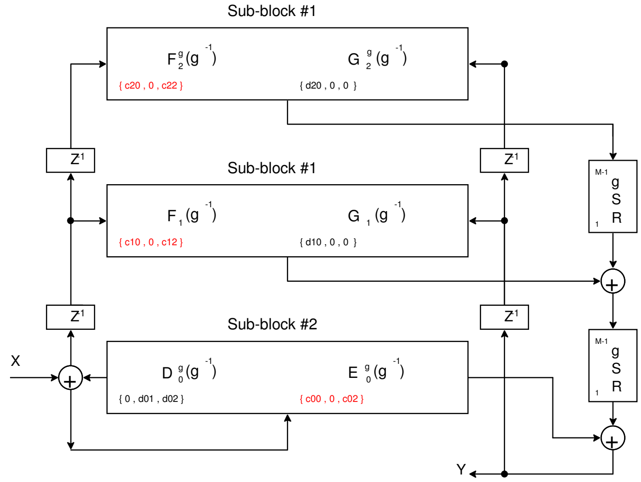

Figure 8. Quadrantal symmetry framework based on framework B

IX. CONCLUSION

New 2-D delta operator VLSI filter structures without global broadcast are presented. They are obtained using 1-D filter sub-blocks with different interconnection frameworks. These structures can realize general 2-D IIR transfer functions, IIR transfer functions with separable denominators, and transfer functions with quadrantal magnitude symmetry. The quadrantal symmetry structure has the advantage of lowest number of multipliers.

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